SMALL DISTANCE CODES FOR HEADER ERROR DETECTION IN ATM NETWORKS

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Abstrak

On transmission links with relatively low bit error rates, error detection codes that have been widely used have d_{min} = 3. These codes may be unneccesarily powerful for fibre links because they require an excessive number of redundant parity check bits. For the purpose of error detection in ATM networks using optical fibre links, some high-rate, small distance codes with d_{min} = 2 may be appropiate. These codes, called Small Distance Codes, can be used to detect all single-error patterns and a large fraction of double errors and other patterns. This paper describes the application of small distance codes for error detection in the ATM cell header. Performances analysis are given and compared to the 8-bit CRC recommended by ITU-T. Three alternatives are examined and analysed. The results are displayed and compared to the standard 8-bit CRC and to 10-bit CRC for ATM network as well.

Keywords: *Small Distance Codes, Header Error, ATM Networks*

Introduction

It should be noted that ATM (Asynchronous Transfer Mode) networks use optical fibre transmission media. This fibre-basd media has a good Bit Error Rate (BER) value (≤ 10^{-8}). For future schemes, the number of parity bits could be reduced (from the current 8 bit CRC header error control recommended by ITU-T) and the number of payload bits increased (for some increase in cell misdelivery rate). Some high rate codes, called small distance codes may therefore be appropriate. Moreover, some of the small distance coding schemes have a small decoding complexity, which will reduce the cost of hardware. This paper investigates the use of Small Distance Codes for header error detection in ATM networks. The study is apllied for short distance block code and small distance cyclic code as well.

The paper begins with an overview of small distance such as characteristics and structure of the codes, constructing the parity check characteristic, and how the error detection circuit implemented for the purpose of the decoding process. Afterward, it describes the application of small distance codes for error detection in the ATM cell header. Performances analysis are given and compared to the 8-bit CRC recommended by ITU-T.

Small Distance Codes

On transmission links with relatively low bit error rates, error detection codes that have been widely used have d_{min} ³ 3. These codes may be unneccesarily powerful for fibre links because they require an excessive number of redundant parity check bits. For the purpose of error detection in ATM networks using optical fibre links, some high-rate, small distance codes with d_{min} = 2 may be appropiate. These codes, called Small Distance Codes, can be used to detect all single-error patterns and a large fraction of double errors and other patterns (M.J. Miller, 1989).

Consider an (n,k) linear block code for which the number of pariry check bits (n,k) is chosen such that:

$$2^{n-k} - 1 < n \quad (1)$$

From (S. Lin , D.J. Costello, 1983) the probability that the code will fail to detect an error in the received block for a Binary Symmetric Channel (BSC) is

$$p(e) = \sum_{i=d_{min}}^{n} A_i \, p^i \, (1-p)^{n-i}$$

A_i= weight spectrum
p = channel error probability
For p<< 1, a small distance code (n,k) is defined for which $A_1 = 0$ and A_2 is minimised (M.J. Miller, 1989).

$$p(e) \approx A_2 \, p^2 \, (1-p)^{n-2} \quad (2)$$

$$A_2 = n - (2^{n-k} - 1) (r - \frac{r(r-1)}{2}) \quad (3)$$

r is the number of times that non-zero (n-k) tuple columns in parity check matrix H are repeated.

$$r = \left\lfloor \frac{n}{2^{n-k} - 1} \right\rfloor \quad (4)$$

$\lfloor x \rfloor$ denotes the largest integer smaller than x.
The parity check matrix (H) of small-distance codes for $A_1 = 0$ and minimised A_2 should have:

1. No column of H containing all zeros.
2. No row of H containing all ones.
3. The numbers of identical column of H should be minimised.
4. The columns of parity check matrix H consist of all non-zero (n-k) tuples repeated r times.
5. There are l other non-zero vectors
   $l = n - r (2^{n-k} - 1)$

One of the notable features of such high speed hardware is parallel encoding and decoding. A parallel circuit can maintain high rates of throughput and is not too complex to implement. An error detection circuit for a Small Distance Code must performs the following functions (M.J. Miller, S. Ahamed, 1986)

1. When the block code is received, check whether it is a valid codeword in the set of 2^k possible codewords.
2. If it is not a valid codeword, then errors have ocuured and are detected.
3. If it is a valid codeword, then either no errors have occured or the error pattern is undetectable by the code.

Small Distance Codes for Header Error Detection

For k =32, we can have several Small Distance Codes which fulfill the requirements of error detection code. We calculate and tabulate the characteristics of Small Distance Code in Table 1. While Figure.1 shows the probability of the undetected errors of the codes.

Table 1
Characteristics of small distance code for ATM cell header (k = 32)

(n,k)	$r = \left\lfloor \frac{n}{2^{n-k} - 1} \right\rfloor$	$A_2 = n - (2^{n-k} - 1) \times (r - \frac{r(r-1)}{2})$	$p(e) \approx A_2 \, p^2 \, (1-p)^{n-2}$
(34,32)	11	166	$166p^2 - 5132p^3$
(35,32)	5	70	$70p^2 - 2130p^3$
(36,32)	2	17	$17p^2 - 578p^3$
(37,32)	1	6	$6p^2 - 210p^3$

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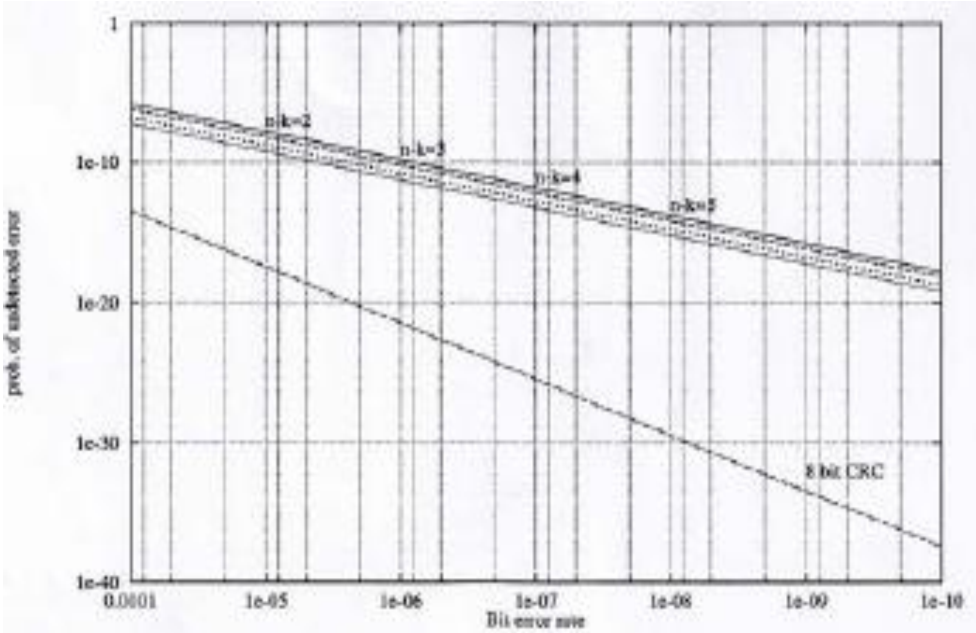

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Figure 1
Probability of undetected error for Small Distance Codes (k=32)

These Small Distance Error detection codes have error consequences as shown in Figure 1. From this figure, we can calculate the performances of each of the code. For an (n,k) Small Distance Code, we have the probability of correct delivery, $P_{corr\text{-}del}$, is the same as probability of no error

$P_{corr\text{-}del} = P_{no\text{-}err} = (1-p)^n$ (5)
$= (1-p)^{34}$ for (34,32) code.
$= (1-p)^{35}$ for (35,32) code.
$= (1-p)^{36}$ for (36,32) code.
$= (1-p)^{37}$ for (37,32) code.

The probability of cell misdelivery is the probability of an undetected error which is not assigned by the VCI (Virtual Channel Identifier). We assume that 50% of these undetected errors will be assigned by the VCI.

$P_{cell\text{-}miss} = 0.5\ P_{un\text{-}det}$
$\approx 0.5 \times (166p^2 - 5132p^3)$ for n = 34 and p<<1
$\approx 0.5 \times (70p^2 - 2130p^3)$ for n = 35 and p<<1
$\approx 0.5 \times (17p^2 - 578p^3)$ for n = 36 and p<<1
$\approx 0.5 \times (6p^2 - 210p^3)$ for n = 37 and p<<1

An undetected error which is not assigned by the VCI will be discarded. Probability of discarded cell is the sum of the probability of detected error and the probability of undetected error which is not assigned by VCI.

Probability of detected error is :
$P_{det} = 1 - P_{no\text{-}err} - P_{undet}$
$P_{disc} = P_{det} + 0.5\ P_{undet}$ (6)
$= 1 - P_{no\text{-}err} - 0.5\ P_{undet}$
$\approx 1 - (1-p)^{34} + 0.5\ (166p^2 - 5132p^3)$ for n = 34 and p<<1
$\approx 1 - (1-p)^{35} + 0.5\ (70p^2 - 2130p^3)$ for n = 35 and p<<1
$\approx 1 - (1-p)^{36} + 0.5\ (17p^2 - 578p^3)$ for n = 36 and p<<1
$\approx 1 - (1-p)^{37} + 0.5\ (6p^2 - 210p^3)$ for n = 37 and p<<1

The probability of discarded cell and cell misdelivery for Small Distance Codes with (n-k) = 2 to 5 are shown in Figures 2 and 3, respectively. While the performance of (36,32) Small Distance Codes is represented in Figure 4.

As shown, probability of discarded cell of several Small Distance Codes (k=32) have the same value as the 8-bit CRC, but it has less value in probability of cell misdelivery. Therefore, these error-detection Small Distance Code is not suitable for services which allow cell misdelivery .

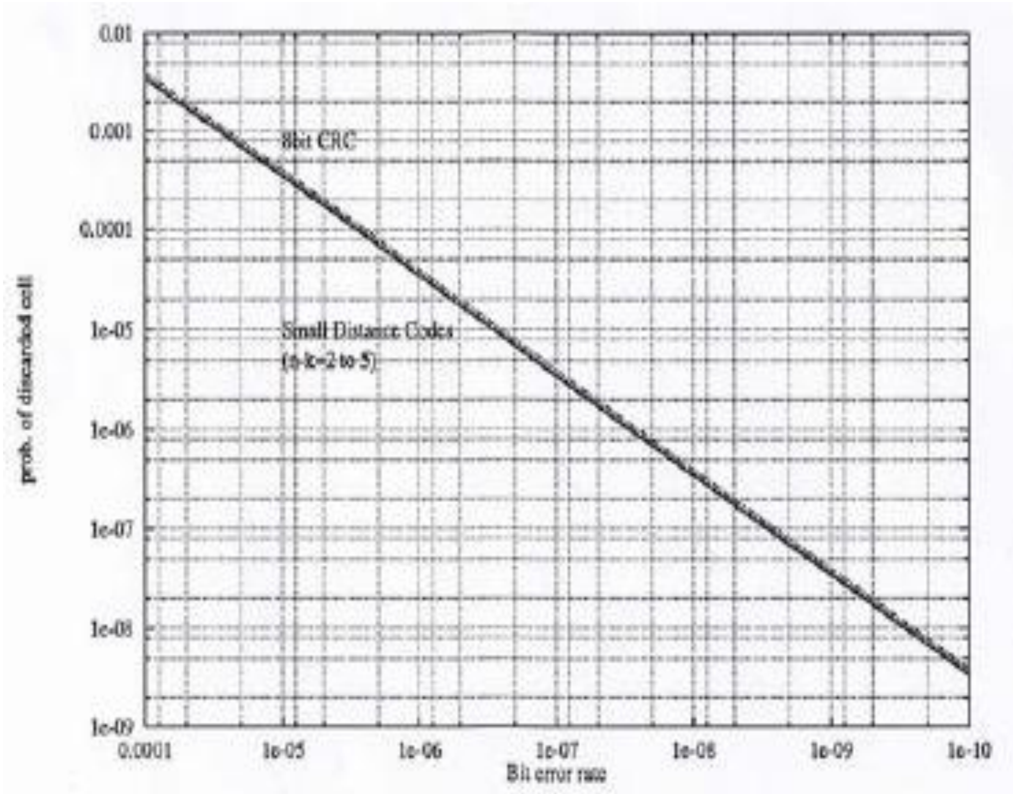

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Figure 2
Probability of discarded cell for Small Distance Codes

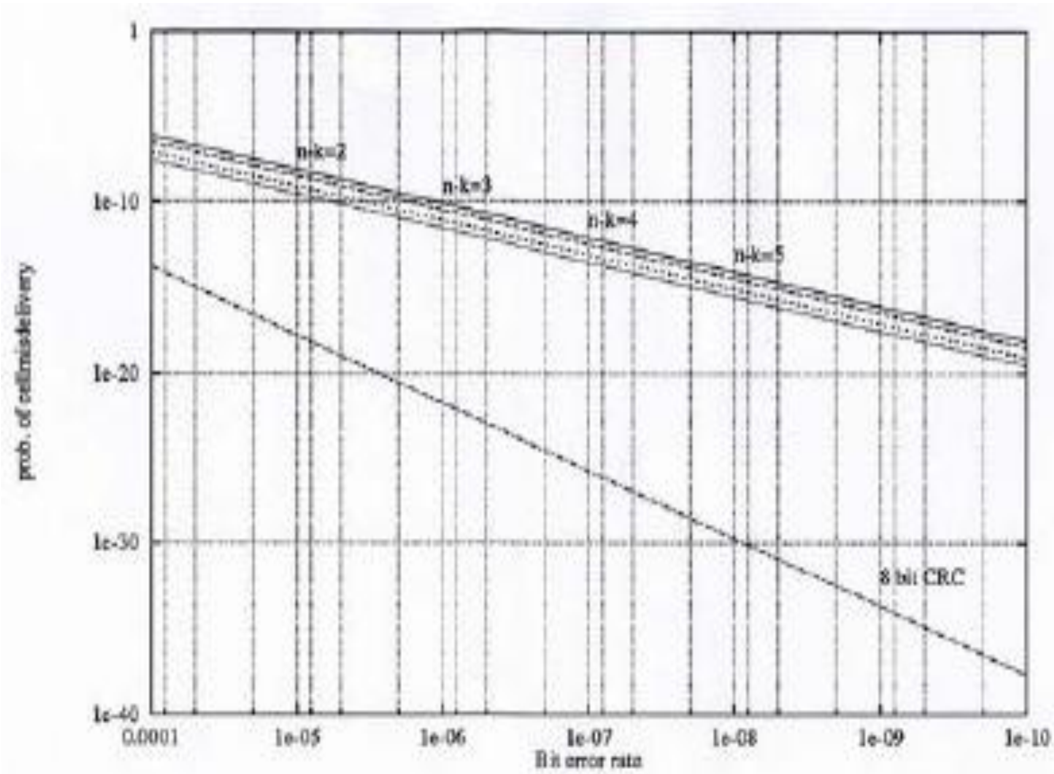

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Figure 3
Probability of Cell missdelivery for Small Distance Codes (k=32)

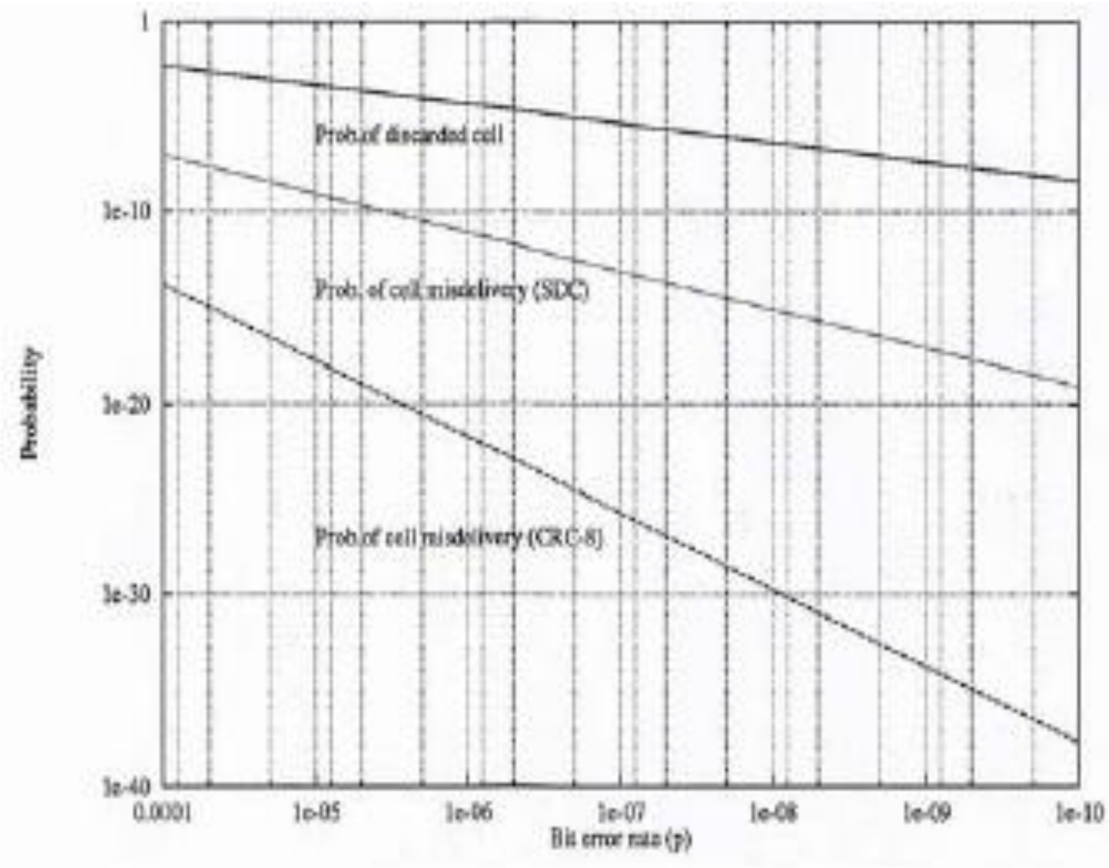

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Figure 4
Performance of (36,32) Small Distance Code

Small Distance Cyclic Code For ATM Networks

For any (n,k) Small Distance Block Code there exists a Small Distance Cyclic Code with generator polynomial g(X) if g(X) divides ($X^n \oplus 1$) and the highest degree of g(X) is (n-k).

$$g = \frac{X^n \oplus 1}{X^k \oplus .... \oplus 1} \qquad (7)$$

$$g = a_{n-k}X^{n-k} \oplus a_{n-k-1}X^{n-k-1} \oplus .... \oplus a_o X^o$$

where, $a_{n-k} = a_o = 1$

$a_{n-k-i} \in \{0,1\}$ for $0 < i < (n-k)$

Small Distance Cyclic Codes are easier to encode and syndrome compute the syndrome in comparison to Small Distance Block Codes (by employing LFSR, Linear Feedback Shift Registers). Using Small Distance Cyclic codes for error detection will result in a simpler hardware implementation.

For the (36,32) Small Distance Code, there exists a (36,32) Small Distance Cyclic Code with $g = X^4 \oplus X^3 \oplus X \oplus 1$ since
$X^{36} \oplus 1 = (X^4 \oplus X^3 \oplus X \oplus 1)$ x $(X^4 \oplus X^2 \oplus 1)$ x $(X^{28} \oplus X^{27} \oplus X^{25} \oplus X^{24} \oplus X^{16} \oplus X^{15} \oplus X^{13} \oplus X^{12} \oplus X^4 \oplus X^3 \oplus X \oplus 1)$

To calculate the probability of this cyclic code, we can use the Mac William's identity of it's dual code. The generator matrix of the dual code is g_2 = h(x) where h(x) is the parity check polynomial.

$$g_2 = \frac{X^{36} \oplus 1}{(X^4 \oplus X^3 \oplus X \oplus 1)}$$

$$= X^{32} \oplus X^{31} \oplus X^{30} \oplus X^{26} \oplus X^{25} \oplus X^{24} \oplus X^{20} \oplus X^{19} \oplus X^{18} \oplus X^{14} \oplus X^{13} \oplus X^{12} \oplus X^8 \oplus X^7 \oplus X^6 \oplus X^2 \oplus X \oplus 1$$

Table 2 gives the calculation of the hamming Weight of the dual (36,4) code. From the table we obtain the weight distribution

$$B_i = \begin{cases} 0 \; ; \text{all } i \text{ except } 0, 12, 18, 24, \text{ and } 36 \\ 1 \; ; i = 0 \text{ and } 36 \\ 3 \; ; i = 12 \text{ and } 24 \\ 8 \; ; =18 \end{cases}$$

From (S. Lin, D.J. Costello, 1983) the probability of undetected error for the (36,32) cyclic code is

$$P_{undet} = 2^{-(n-k)} \left(\sum_{i=0}^{n} B_i (1-2p)^i \right) - (1-p)^n \quad (10)$$

$$= 2^{-4}\{1 + 3(1-2p)^{12} + 8(1-2p)^{18} + 3(1-2p)^{24} + (1-2p)^{36}\} - (1-p)^{36}$$

The probability of the undetected error is seen in Figure 5 comparison to the 8-bit CRC code recommended by CCITT and (36,32) Small Distance Block Code.

Similar to the calculation for the Small Distance Error Detection code in section 2, we can compute the probability of cell misdelivery and discarded cell of this Small Distance Cyclic Code for n = 36 and (n-k) = 4.

$$P_{cell\text{-}miss} = 0.5\, P_{undet}$$
$$= 0.5 \times 2^{-4}\{1 + 3(1-2p)^{12} + 8(1-2p)^{18} + 3(1-2p)^{24} + (1-2p)^{36}\} - 0.5 \times (1-p)^{36}$$

$$P_{cell\text{-}disc} = P_{det} + 0.5\, P_{undet}$$
$$= (1 - P_{no\text{-}error} - P_{undet}) - 0.5\, P_{undet}$$
$$= 1 - P_{no\text{-}error} - 0.5\, P_{undet}$$
$$= 1 - (1-p)^{36} - 0.5 \times 2^{-4}\{1 + 3(1-2p)^{12} + 8(1-2p)^{18} + 3(1-2p)^{24} + (1-2p)^{36}\} + 0.5 \times (1-p)^{36}$$

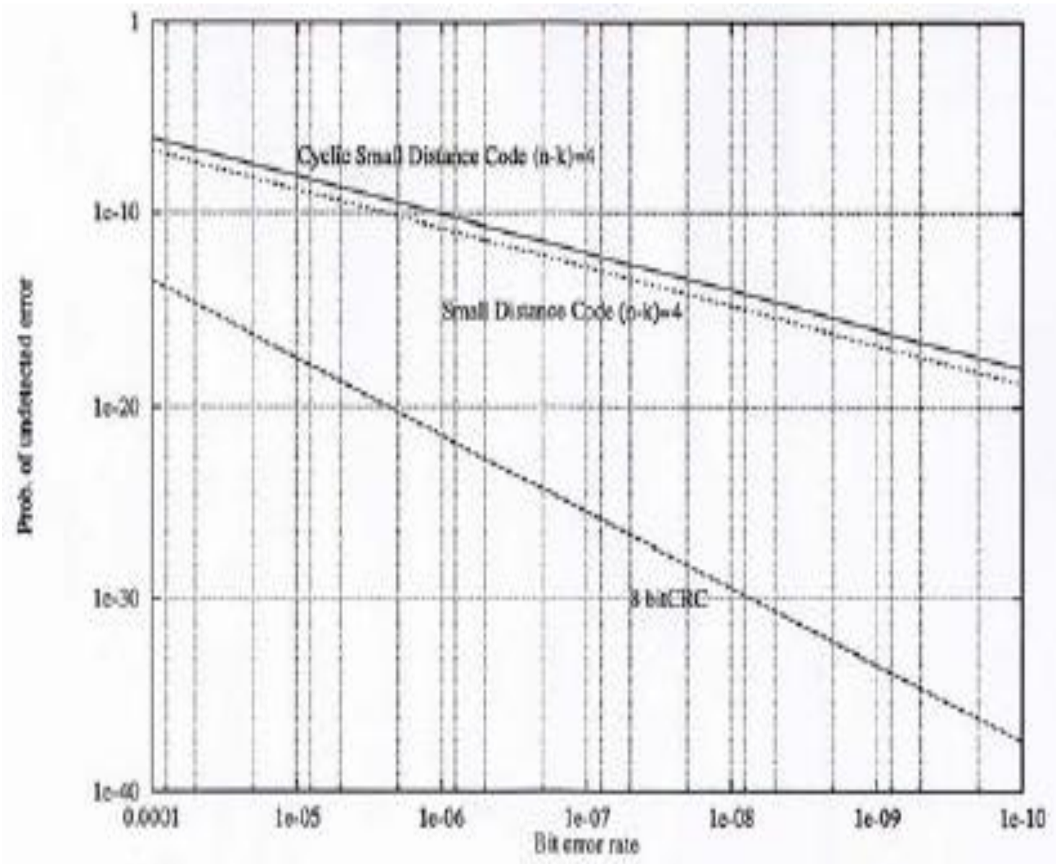

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Figure 5
Probability of undetected error for Small Distance Cyclic Code (k=32)

Table 4
Hamming weights of the dual code of (36,32) Small Distance Code

INPUT POLYNOMIAL	HAMMING WEIGHT
0	0
1	18
X	18
$X \oplus 1$	12
X^2	18
$X^2 \oplus 1$	24
$X^2 \oplus X$	12
$X^2 \oplus X \oplus 1$	18
X^3	18
$X^3 \oplus 1$	36
$X^3 \oplus X$	24
$X^3 \oplus X \oplus 1$	18
$X^3 \oplus X^2$	12
$X^3 \oplus X^2 \oplus 1$	18
$X^3 \oplus X^2 \oplus X$	18
$X^3 \oplus X^2 \oplus X \oplus 1$	24

Source: Self Compiled

The probability of discarded cell and cell misdelivery, compared to 8-bit CRC code and (36.32) Small Distance Block Code, are shown in Figure 6 and 7, respectively. Figure 8 gives the code performance of Small Distance Cyclic Code for k = 32. Interesting result come out from Figure 6. All the codes share the same value on the probability of discarded cell. For the probability of cell misdelivery, 8-bit CRC gives the best result, but comparing the result of Small Distance Cyclic Code to Small Distance Block Code, the difference in the cell misdelivery is very insignificant.

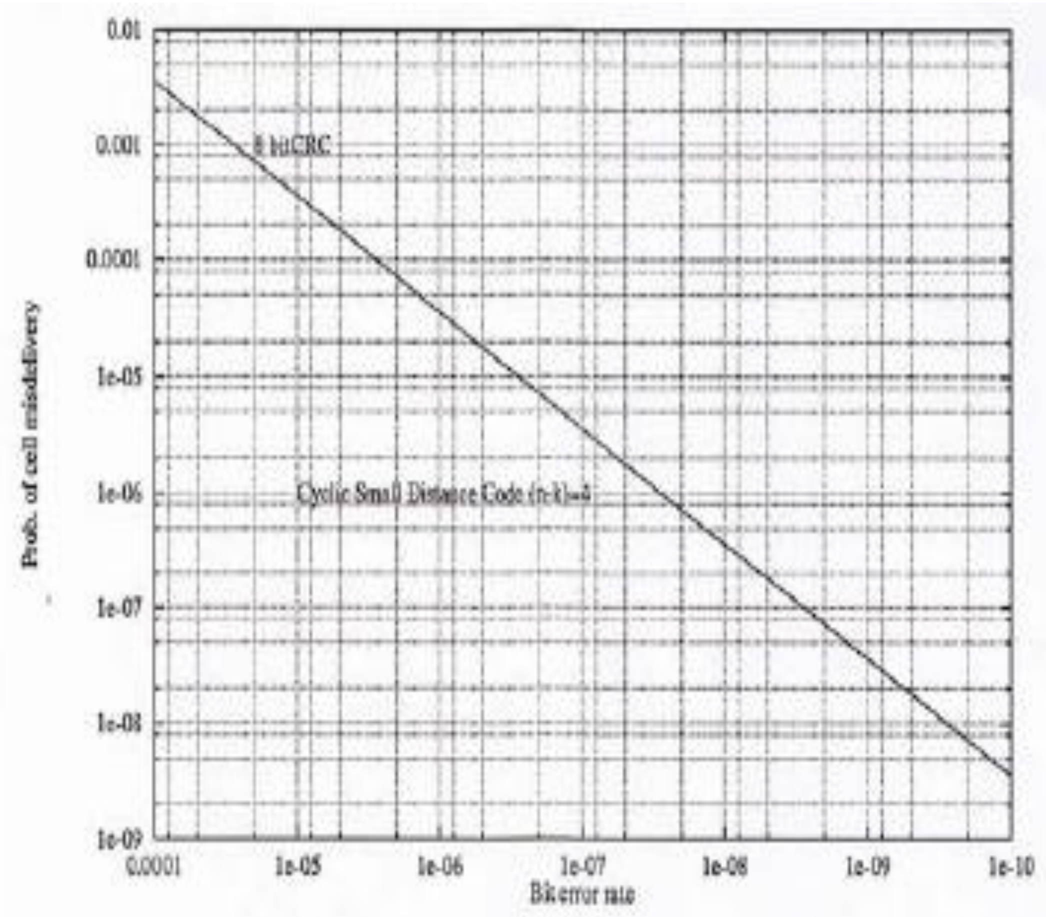

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Figure 6
Probability of Discarded cell for Small Distance Cyclic Code (k=32)

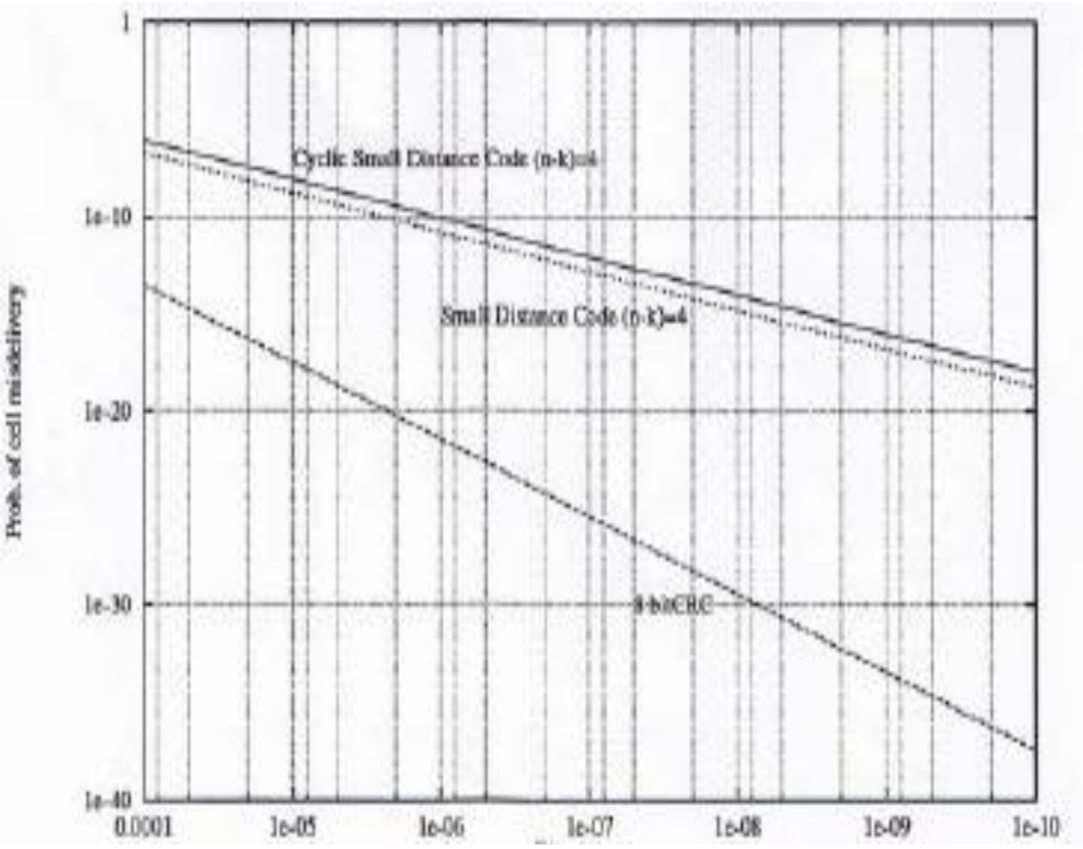

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Figure 7
Probability of cell missdelivery for Small Distance Cyclic Code (k=32)

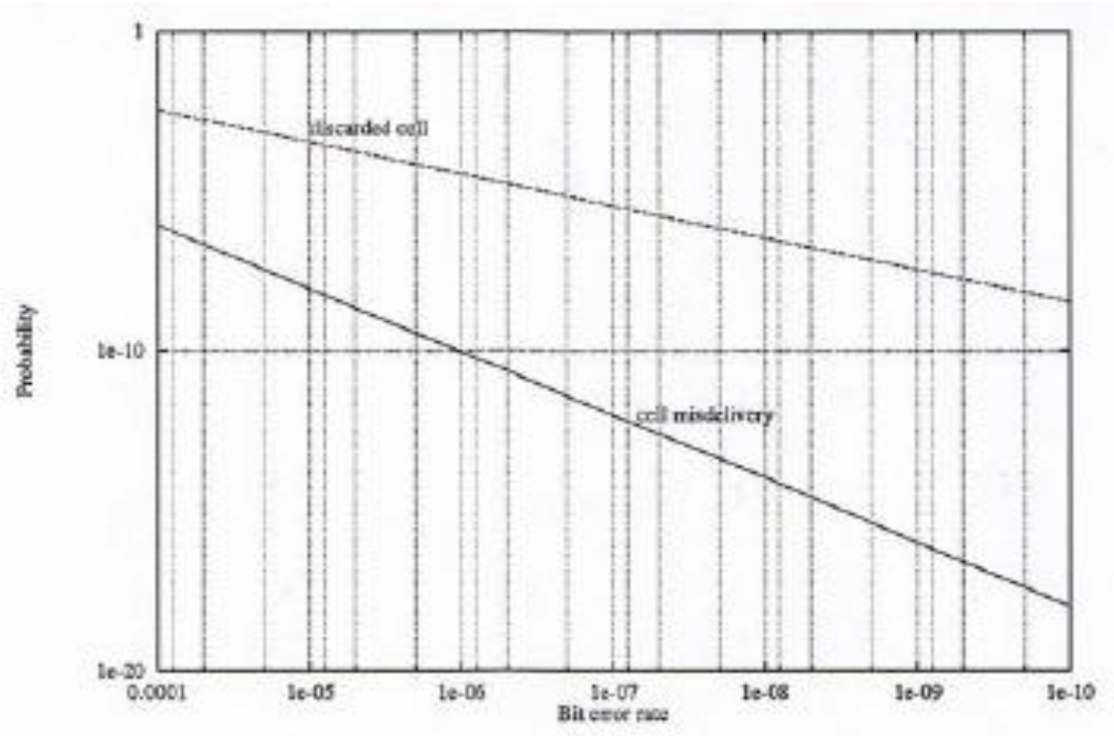

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Figure 8
Performance of (36,32) Small Distance Cyclic Code

Conclusion

Small Distance Codes, which can detect all single error patterns and a large number of double-error and other patterns, are a good alternative for error detection codes in ATM networks. It uses parity check bits without reducing the discarded cell rate. For some Small Distance Codes, there exists Small Distance Cyclic Codes. These Small distance Cyclic Codes have similar performances to the Small Distance Codes but have advantages in hardware implementation. Both Small Distance Block Codes and Cyclic Codes need less parity bits compare to the 8-bit and 10-bit CRC codes. Less parity can lead to less overhead and more data can be sent.

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